EQUIVARIANT HOMOTOPY THEORY: PROBLEM SET 3

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(1) Suggested reading [Str09] for CGWH spaces and [Hov99, §2.4, §4.1,§4.2].

(2) Show that the periodic $\mathbb{F}_2[C_2]$-complex :

$$T = \cdots \to \mathbb{F}_2[C_2] \xrightarrow{1+g} \mathbb{F}_2[C_2] \xrightarrow{1+g} \mathbb{F}_2[C_2] \to \cdots$$

is acyclic and projective in every degree, but $0 \to T$ does not have the LLP wrt $\mathbb{F}_2[C_2] \xrightarrow{\epsilon} \mathbb{F}_2$.

(3) Prove the following lemma from [Hov99, Lemma 4.2.2]: Suppose that $\mathscr{C}, \mathscr{D}, \mathscr{E}$ are model categories and $(\otimes, \hom_r, \hom_l, \phi_r, \phi_l)$ is an adjunction of two variables $\mathscr{C} \times \mathscr{D} \to \mathscr{E}$. Show the following are equivalent:

  (i) $\otimes$ is a Quillen bifunctor.

  (ii) Given a cofibration $g \in \mathscr{D}(W,X)$ and a fibration $p \in \mathscr{E}(Y,Z)$, the induced map

  $$\hom_{r,\Box}(g,p)\colon \hom_r(X,Y) \to \hom_r(X,Z) \times_{\hom_r(W,Z)} \hom_r(W,Y)$$

  is a fibration in $\mathscr{C}$ that is trivial if either g or p is.

  (iii) Given a cofibration $f \in \mathscr{C}(U,V)$ and a fibration $p \in \mathscr{E}(Y,Z)$, the induced map

  $$\hom_{l,\Box}(f,g)\colon \hom_l(V,Y) \to \hom_l(V,Z) \times_{\hom_l(U,Z)} \hom_l(U,Y)$$

  is a fibration in $\mathscr{D}$ that is trivial if either f or p is.

  Note that this is easy once one verifies the adjointness of two diagrams. Unfortunately this part is quite tedious and requires tabulating all of the data.

(4) ([May99, Ch. 5] Show that

  (i) Any subspace of a weak Hausdorff space is weak Hausdorff.

  (ii) Any closed subspace of a k-space is a k-space.

  (iii) An open subset U of a CGWH X is CGWH if each point $x \in U$ has an open neighborhood in X with closure contained in U.

  (iv) A space is Tychonoff (points are closed, and for each point $x \in X$ and closed subset A not containing x there is a continuous function $f: X \to I$ such that $f(x) = 0$ and $f(A) = 1$) if and only if it can be embedded in a cube.

  (v) There are Tychonoff spaces that are not k-spaces, but every cube is a compact Hausdorff space.

  (vi) In view of the above, what should a subspace of CGWH-space be?

(5) Suppose G is a topological group and H is an open subgroup, show that H is closed.

References

[Hov99] Mark Hovey, *Model categories*, Mathematical Surveys and Monographs, vol. 63, American Mathematical Society, Providence, RI, 1999. MR 1650134 (99h:55031)

[May99] J. P. May, *A concise course in algebraic topology*, Chicago Lectures in Mathematics, University of Chicago Press, Chicago, IL, 1999. MR MR1702278 (2000h:55002)

[Str09] N. Strickland, *The category of cgwh spaces*, Available from Strickland's website, 2009.