EQUIVARIANT HOMOTOPY THEORY: PROBLEM SET 4

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(1) Suggested reading: [Bor94a, §6] and [Hov99, §4.1] for monoidal categories and enriched categories, [May96, §1.3-1.4] and [tD87, §2.1] for G-CW complexes and Bredon homology/cohomology.

(2) [Bor94b, §6] Let $\mathbb{R}_+$ be the non-negative real numbers and let $\underline{\mathbb{R}}_+ = \mathbb{R}_+ \cup \{\infty\}$. Show that $\underline{\mathbb{R}}_+$ is a closed bicomplete symmetric monoidal category: where

  (i)
$$\underline{\mathbb{R}}_+(r,s) = \begin{cases} * & \text{if } r \geq s, \\ \emptyset & \text{if } r < s. \end{cases}$$
  (ii) $r \otimes s = r + s$
  (iii) $\hom(r,s) = \max(s-r,0)$.

(3) Show that the metric on a metric space X can make X into a $\underline{\mathbb{R}}_+$-enriched category.

(4) Verify the $\mathscr{V}$-enriched Yoneda lemma: Show that if $F\colon A^{\mathrm{op}} \to B$ is a functor of $\mathscr{V}$-enriched monoidal categories then
$$B^{A^{\mathrm{op}}}(A(-,a),F) \cong F(a).$$

(5) Construct G-CW structures on the representation spheres $S(V)$ for
  (i) $G = C_n = \langle g \rangle$ and $V = \mathbb{C}$ with $gv = e^{2\pi i/n}v$.
  (ii) $G = S_3$ thought of as the group of isometries of an isoceles triangle and V is obtained by taking embedding an isoceles triangle with center at the origin of $V = \mathbb{R}^2$, and extending the rotations and reflections to V.
  (iii) $G = \mathbb{T}$ (the circle group) acting via the standard embedding $\mathbb{T} \to \mathbb{C}^* \to \mathbb{C} = V$.

References

[Bor94a] Francis Borceux, *Handbook of categorical algebra*, vol. 2, Cambridge University Press, 1994.

[Bor94b] ______, *Handbook of categorical algebra*, vol. 1, Cambridge University Press, 1994.

[Hov99] Mark Hovey, *Model categories*, Mathematical Surveys and Monographs, vol. 63, American Mathematical Society, Providence, RI, 1999. MR 1650134 (99h:55031)

[May96] J. P. May, *Equivariant homotopy and cohomology theory*, CBMS Regional Conference Series in Mathematics, vol. 91, Published for the Conference Board of the Mathematical Sciences, Washington, DC, 1996, With contributions by M. Cole, G. Comezaña, S. Costenoble, A. D. Elmendorf, J. P. C. Greenlees, L. G. Lewis, Jr., R. J. Piacenza, G. Triantafillou, and S. Waner. MR MR1413302 (97k:55016)

[tD87] Tammo tom Dieck, *Transformation groups*, de Gruyter Studies in Mathematics, vol. 8, Walter de Gruyter & Co., Berlin, 1987. MR 889050 (89c:57048)