EQUIVARIANT HOMOTOPY THEORY: PROBLEM SET 5

JUSTIN NOEL

(1) Suggested reading: [May96, §1.3-1.4] and [tD87, §2.1] for G-CW complexes and Bredon homology/cohomology.

(2) Consider the following contravariant coefficient systems for C_p:
   (*i*) $M(C_p) = \mathbb{F}_p[C_p]$ (with the C_p action given by translation of basis elements) and $M(C_p/C_p) = \mathbb{F}_p$.
   (*ii*) $N(C_p) = 0$ and $N(C_p/C_p) = \mathbb{F}_p$.
   Show $H^*_{C_p}(X;M) \cong H^*(X;\mathbb{F}_p)$ and $H^*_{C_p}(X;M) \cong H^*(X^G;\mathbb{F}_p)$.

(3) For $H < G$ and an H-space Y, let
$$\mathrm{Ind}_H^G Y = G \times_H Y = G \times Y/\sim$$
where $(gh^{-1}, y) \sim (g, hy)$ for each $h \in H$.
   (*i*) Show that the Ind_H^G is left adjoint to the forgetful functor $\mathrm{Res}_H^G \colon \mathcal{T}op_G \to \mathcal{T}op_H$.
   (*ii*) Find the right adjoint, Coind_H^G, to Res_H^G.
   (*iii*) Find the analogues of these functors in $\mathcal{T}op_{*,G}/\mathcal{T}op_{*,H}$ and $\mathbb{Z}[G]/\mathbb{Z}[H]$-modules.
   (*iv*) Define a natural transformation $\mathrm{Ind}_H^G \to \mathrm{Coind}_H^G$ for each pair of categories in the previous exercise.
   (*v*) Show that in the case of the $\mathbb{Z}[G]/\mathbb{Z}[H]$-module categories, the natural transformation of the previous exercise is an isomorphism. Show that this is not true for $\mathcal{T}op_{*,G}$. What is the specific property that separates these two cases?
   (*vi*) For a G-space X construct an explicit isomorphism
$$\mathrm{Ind}_H^G \mathrm{Res}_H^G X \cong G/H \times X$$
   where G acts diagonally on the right hand side.
   (*vii*) Using the previous exercise and that $\mathcal{T}op_G$ is cartesian closed, show that
$$\mathrm{Coind}_H^G \mathrm{Res}_H^G \cong \mathcal{T}op(G/H, X).$$

REFERENCES

[May96] J. P. May, *Equivariant homotopy and cohomology theory*, CBMS Regional Conference Series in Mathematics, vol. 91, Published for the Conference Board of the Mathematical Sciences, Washington, DC, 1996, With contributions by M. Cole, G. Comezaña, S. Costenoble, A. D. Elmendorf, J. P. C. Greenlees, L. G. Lewis, Jr., R. J. Piacenza, G. Triantafillou, and S. Waner. MR MR1413302 (97k:55016)

[tD87] Tammo tom Dieck, *Transformation groups*, de Gruyter Studies in Mathematics, vol. 8, Walter de Gruyter & Co., Berlin, 1987. MR 889050 (89c:57048)