TOPICS IN HIGHER CATEGORY THEORY

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The goal of this seminar is to learn about various topics in the theory of higher categories. Here is a provisional list of suggested topics.

The list of topics is not meant to be complete; further suggestions are welcome. The list of references is far from comprehensive in many cases, and is also far from being very specific. The speakers are encouraged to find additional references, select a more specific focus for their talks, and discuss it with the organizers.

1. Higher categories and higher topoi

(1) Simplicial categories and Bergner's model structure [Ber07] [Lur09, App. A.3].
(2) The Riehl-Verity approach to higher category theory (1-2 talks).
(3) Marked simplicial sets and (co)cartesian model structures (2 talks) [Lur09, Ch. 3].
(4) Cofinal maps and Joyal's generalization of Quillen's theorem A [Lur09, §4.1].
(5) Adjoint functors. This includes the comparisons between different definitions, proofs of the uniqueness of adjoints and of other basic properties (such as left adjoints preserve colimits). Localization functors can be discussed as special cases [Lur09, §5.2]. Dependencies: (3).
(6) ∞-categories of Ind-objects and Ind-completions [Lur09, §5.3]. Definitions of: Compact objects, categories of Ind-objects and Ind-completions. (1-2 talks) The universal property $\mathrm{Ind}(\mathsf{C})$.
(7) Accessible ∞-categories. Definitions and basic properties [Lur09, §5.4] (1-2 talks). Dependencies: (6).
(8) Universal property of presheaf categories and localizations thereof [Lur09, §5.1.5], [Dug01b]. Dependencies: (7), (5).
(9) Presentable ∞-categories. The ∞-categorical analogue of Simpson's theorem. Relationship to combinatorial model categories [Lur09, §5.5.1-5.5.4, App. A.2], [Dug01a]. Dependencies: (8).
(10) Homotopy algebras [Qui67]. Examples! Projective compact generators. Algebraic theories. [Lur09, §5.5.8-5.5.9], [Ros07].
(11) Homotopical sheaf theory. [DHI04] [Jar96]. Descent vs. hyperdescent [Lur09, §6.5.4]. Relevant history and applications.
(12) Topics in ∞-topoi:
    (a) Giraud's axioms [Lur09, §6.1.1-6.1.5], [Rez10] (2 talks). Dependencies: (9).
    (b) Grothendieck topologies and sheaves [Lur09, §6.2.2-6.2.4]. Dependencies: (11), (8).
    (c) The ∞-category of ∞-topoi: geometric and étale morphisms, (co)limits [Lur09, §6.3].
    (d) The proper base change theorem [Lur09, §7.3].
(13) Models for (∞, n)-categories and comparisons: n-fold Segal spaces, Θ_n-spaces,....

(14) The axiomatic approaches of Toën and Barwick-Schommer-Pries to higher categories.

2. Higher algebra and related topics

(15) Stable ∞-categories. Basic theory, triangulated categories, compact generation. [Lur14, §1]
(16) Stabilization of a presentable ∞-category and the associated universal property. The universal property of spectra [Lur14].
(17) The Dold-Kan equivalence for stable ∞-categories [Lur14].
(18) Universal property of non-negatively graded chain complexes [Lur14]. Dependencies: (8).
(19) Introduction to (symmetric-)monoidal ∞-categories [**?**], [Lur14]. This talk is to present some simple cases of ∞-operads (although we will not know what an ∞-operad is yet!), namely monoidal and symmetric monoidal ∞-categories. These can be defined in several ways. One of the most minimal is as functors from Δ^{op} (resp. $\mathrm{FinSets}_*$) to ∞-categories which satisfy a Segal condition. Alternatively, via unstraightening, they can be defined as cocartesian fibrations over these categories satisfying a corresponding Segal condition.
(20) Introduction to the theory of ∞-operads [Lur14]. Simplicial operads, multicategories, and ∞-operads. The operadic coherent nerve functor. Dependencies: (5).
(21) A survey of the theory of dendoridal sets. Moerdijk et. al.
(22) The theory of E_n-algebras. The little n-cubes/disks operad and the associated ∞-operad. Topics: colimits, limits, E_n-ring spectra, Toda brackets, 'power' operations. Dependencies: (6).
(23) Module categories over E_n-algebras. Definitions and properties. Dependencies: (8).
(24) Dunn's additivity theorem.
(25) The symmetric monoidal structure on presentable ∞-categories and the Day convolution product.
(26) Monoidal localizations and colocalizations of $\mathcal{O}$-monoidal ∞-categories.
(27) Lurie's extension of the Barr-Beck theorem to ∞-categories.

3. More Advanced topics

The subject areas below are huge. One can either try to present a very limited introduction to the subject or try to gather a group of people together to do a block of lectures.

(28) Goodwillie calculus in the setting of ∞-categories.
(29) The ∞-categorical approach to Thom spectra and orientation theory.
(30) Picard groupoids of symmetric monoidal ∞-categories. References: Stojanoska-Mathew and Heard-Stojanoska-Mathew.
(31) The Baez-Dolan cobordism hypothesis after Lurie.
(32) Universal property of algebraic K-theory. Dependencies (8), (9),(1). Papers by Tabuada, Blumberg-Gepner-Tabuada, and Barwick.
(33) Rational homotopy theory. Quillen. Sullivan. Lurie.
(34) p-adic homotopy theory. Mandell. Lurie.

References

[Ber07] Julia E. Bergner. A model category structure on the category of simplicial categories. *Trans. Amer. Math. Soc.*, 359(5):2043–2058, 2007.

[DHI04] Daniel Dugger, Sharon Hollander, and Daniel C. Isaksen. Hypercovers and simplicial presheaves. *Math. Proc. Cambridge Philos. Soc.*, 136(1):9–51, 2004.

[Dug01a] Daniel Dugger. Combinatorial model categories have presentations. *Adv. Math.*, 164(1):177–201, 2001.

[Dug01b] Daniel Dugger. Universal homotopy theories. *Adv. Math.*, 164(1):144–176, 2001.

[Jar96] J. F. Jardine. Boolean localization, in practice. *Doc. Math.*, 1:No. 13, 245–275 (electronic), 1996.

[Lur09] Jacob Lurie. *Higher topos theory*, volume 170 of *Annals of Mathematics Studies*. Princeton University Press, Princeton, NJ, 2009.

[Lur14] J. Lurie. *Higher Algebra*. Available at `http://www.math.harvard.edu/~lurie/papers/HigherAlgebra.pdf`, September 2014. http://www.math.harvard.edu/ lurie/papers/HigherAlgebra.pdf.

[Qui67] Daniel G. Quillen. *Homotopical algebra*. Lecture Notes in Mathematics, No. 43. Springer-Verlag, Berlin, 1967.

[Rez10] Charles Rezk. Toposes and homotopy toposes. February 2010.

[Ros07] J. Rosický. On homotopy varieties. *Adv. Math.*, 214(2):525–550, 2007.