TOPICS IN HIGHER CATEGORY THEORY

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The purpose of this seminar is to study the general theory of higher categories and its applications. Higher category theory provides a powerful language for handling the complexity of encoding relations, relations between relations, and "so on". In addition to conceptualizing classical results by placing them in a more general context, they have proven essential for studying homotopy theories themselves.

There will be talks on foundational topics (Talks 2-5 and 9), on special classes of ∞-categories (Talks 6-7, 10, 12-13), and talks on examples and applications (Talks 8 and 11-12).

Participants should have some familiarity with the theory of ∞-categories.

1. SCHEDULE OF TALKS

- **Talk 1** (11.4.2016): Justin Noel and Georgios Raptis, *Introduction.*

- **Talks 2** (18.4.2016) **and 3** (25.4.2016): Daniel Schäppi, *The Riehl-Verity approach to higher category theory.* Suggested reading: [RV, RV15].

- **Talk 4** (2.5.2016): Hoang Kim Nguyen, *Cartesian fibrations.* References: [Lur09, 2.4], [MG].

- **Talk 5** (9.5.2016): Thomas Fiore, *Adjoint functors between ∞-categories.* Description: This includes the comparisons between different definitions, proofs of the uniqueness of adjoints and of other basic properties (such as left adjoints preserve colimits). Localization functors can be discussed as special cases. Suggested reading: [Lur09, 5.2].

- **Talks 6** (23.5.2016) **and 7** (30.5.2016): Christoph Schrade, *Presentable ∞-categories.* Description: Basic definitions of accessible ∞-categories, categories of Ind-objects, and Ind-completions. The universal property of the presheaf category could be mentioned. The ∞-categorical analogue of Simpson's theorem. References: [Lur09, 5.1.5, 5.3-5.5.4, App. A.2].

- **Talk 8** (6.6.2016): Koen van Woerden, *Homotopy algebras.* Description: Projective compact generators, algebraic theories. Examples: Simplicial (abelian) groups, (commutative) rings, Lie algebras,.... Potentially: Quillen's definition of homology and cohomology for objects in such categories. Suggested reading: [Qui67], [Lur09, 5.5.8-5.5.9], [Ros07].

- **Talk 9** (13.6.2016): Justin Noel, *Marked simplicial sets and (co)cartesian model structures.* References: [Lur09, Ch. 3].

- **Talk 10** (20.6.2016): Oriol Raventós, *Introduction to (symmetric-)monoidal ∞-categories.* Description: The basic definitions and properties of monoidal and symmetric monoidal ∞-categories. References: [Gro10], [Lur14, §2.1.1-2.1.3].

- **Talk 11** (27.6.2016): Hoang Kim Nguyen, *The ∞-categorical approach to Thom spectra and orientation theory.* New perspectives on May's generalized theory of Thom spectra and orientations. References: [ABG$^+$08, ABG$^+$14b, ABG$^+$14a, May77].

- **Talk 12** (4.7.2016): Christoph Eibl, *Homotopical sheaf theory.* Description: A basic introduction to the model categories of simplicial (pre)sheaves and their properties. Discussion of different types of descent. Relevant history and applications. Suggested reading: [DHI04], [Jar96], [Lur09, §6.5.4], [Rez].

- **Talk 13** (11.7.2016): Georgios Raptis, *∞-topoi.* Description: Giraud axioms, Grothendieck topologies and sheaves. References: [Lur09, Ch. 6], [Rez].

References

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[RV15] Emily Riehl and Dominic Verity. The 2-category theory of quasi-categories. *Adv. Math.*, 280:549–642, 2015.