LIE THEORY: PROBLEM SET 3

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1. Suggested reading the relevant parts of [Lee03, Ch. 3,5,12-14] .
2. Show that if G is an abelian Lie group then the Lie algebra associated to G is abelian. (Hint: Given a smooth function f on G and a left invariant derivation $\tilde{X}$ associated to $X \in T_eG$, show that $\tilde{X}f$ is the smooth function which at g is $d/dt|_{t=0} f(g \cdot \gamma_X(t))$ and $\gamma\colon (-\epsilon,\epsilon) \to G$ satisfies $\gamma(0) = e$ and $\gamma'(0) = X$. Now compute $[X,Y]f$).
3. Identify the Lie algebras of $U(n)$ and $SU(n)$ respectively.
4. Show that every discrete normal subgroup of a connected Lie group is central (hint: consider the map $G \to N, g \mapsto ghg^{-1}$ where $h \in N$).
5. Show that the fundamental group of any Lie group is commutative.
6. Suppose that $F\colon G_1 \to G_2$ is a morphism of connected Lie groups which induces an isomorphism between their Lie algebras $\mathfrak{g}_1 \xrightarrow{\cong} \mathfrak{g}_2$. Show that F is a covering map and $\ker F$ is a discrete central subgroup.

REFERENCES

[Lee03] John M. Lee, *Introduction to smooth manifolds*, Graduate Texts in Mathematics, vol. 218, Springer-Verlag, New York, 2003. MR 1930091 (2003k:58001)