LIE THEORY: PROBLEM SET 4

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1. Suggested reading the relevant parts of [Lee03, Ch. 7] [Lee11, Ch. 7].
2. Find an example of a Lie subgroup H of a Lie group G such that G/H is not a smooth manifold.
3. Let $SO(n) < SO(n+1)$ be a subgroup fixing some non-zero vector v in $\mathbb{R}^{n+1}$. Show that $SO(n+1)/SO(n) \cong S^n$.
4. Construct a Lie group map $\rho\colon S^1 \to U(n)$ splitting the determinant map.
5. Show that $U(n) \cong S^1 \times SU(n)$ as smooth manifolds, but not as Lie groups.
6. Give an example of a smooth proper action of a Lie group on a smooth manifold such that the quotient space is not a topological manifold.
7. Show that $SO(3) \cong SU(2)/\{\pm \mathrm{id}\}$.

REFERENCES

[Lee03] John M. Lee, *Introduction to smooth manifolds*, Graduate Texts in Mathematics, vol. 218, Springer-Verlag, New York, 2003. MR 1930091 (2003k:58001)

[Lee11] ———, *Introduction to topological manifolds*, second ed., Graduate Texts in Mathematics, vol. 202, Springer, New York, 2011. MR 2766102 (2011i:57001)