LIE THEORY: PROBLEM SET 7

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1. Show that the adjoint homomorphism $\mathrm{Ad}\colon SU(2) \to GL_3(\mathbb{R})$ from last week induces an isomorphism of Lie groups $SU(2) \to SO(3)$.
2. Let P_2 be the space of quadratic polynomials in x with coefficients in $\mathbb{R}$.
   2.1. Let $G = \mathbb{R}$ act on P_2 by $t \cdot f(x) = f(x + t)$. Show that this induces a homomorphism of Lie groups $G \to GL_3(\mathbb{R})$.
   2.2. Calculate the induced homomorphism on Lie algebras.
   2.3. Identify the previous homomorphism with the map sending $k\,d/dt$ to $k\,d/dx|_{x=0}$.
   2.4. Use the exponential map to recover the Taylor expansion formula:
   $$f(x+t) = \sum_{n \geq 0} \frac{t\,d/dx|_{x=0}}{n!} f.$$
3. Calculate $\det_*\colon \mathfrak{gl}_n\mathbb{R} \to \mathfrak{gl}_1\mathbb{R}$.
4. Show that $\det(e^M) = e^{\mathrm{tr}M}$.

References