LIE THEORY: REVIEW SHEET

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1. Suggested reading: Lectures 1 + 2 from Hsiang and the relevant sections of Lee.
2. Be prepared to define and have examples involving:
   2.1. Smooth manifolds.
   2.2. Tangent vectors and bundles.
   2.3. Derivations! (Know the relationship to the above)
   2.4. The bracket on derivations and sections of the tangent bundle.
   2.5. Submersions, immersions, embeddings.
   2.6. Lie groups (subgroup, closed subgroup, center, abelian, torus, connected, universal covering).
   2.7. Lie group actions (representations, stabilizer groups, orbits).
   2.8. Left/right invariant vector fields!
   2.9. Lie algebras (and the relationship to the above)! as well as (simple, ideal, subalgebra, commutative).
   2.10. Flows.
   2.11. One parameter subgroups.
   2.12. The exponential homomorphism.
   2.13. Distributions and integral manifolds.
   2.14. Representations (irreducible, reducible, completely reducible).
   2.15. The invariant Haar measure on a Lie group.
3. You should know about
   3.1. The constant rank theorem and friends.
   3.2. The closed subgroup theorem.
   3.3. The fundamental results about covering spaces.
   3.4. The conditions that guarantee that if we are given a manifold M with smooth G-action then M/G admits a manifold structure.
   3.5. Lie's structure theorems relating Lie groups and Lie algebras! Have some idea how these are proven. I will not ask for detailed arguments about this, but it is good to see how the pieces fit together.
   3.6. How to identify the Lie algebras of $SL_n(\mathbb{R})$, $GL_n(\mathbb{R})$, $O(n)$, $SO(n)$, $GL_n(\mathbb{C})$, $SL_n(\mathbb{C})$, $U(n)$, $SU(n)$, Sp_n, $Aut(\mathfrak{g})$...!
   3.7. How to calculate the homomorphism $F_* : Lie(G_1) \to Lie(G_2)$ from an explicit homomorphism $F : G_1 \to G_2$ of Lie groups!
   3.8. The Baker-Campbell-Hausdorff formula.
   3.9. The properties of the exponential homomorphism including how it acts in the case of matrix groups!
   3.10. The relationship between $Ad : G \times G \to G$ and $Ad : G \to GL(\mathfrak{g})$ and $ad : \mathfrak{g} \to GL(\mathfrak{g})$.

3.11. How one shows that the every finite dimensional representation of a compact Lie group is completely reducible.

4. With respect to potential problems you should be prepared for:
   4.1. Anything from the homework (although you will only have to do a part or a simplified version of longer problems)!
   4.2. Questions about low dimensional Lie algebras: classify 1+2 dimensional Lie algebras show that $\mathfrak{su}_2$ is simple, etc!
   4.3. Showing the fundamental group of a Lie group is abelian.
   4.4. Calculating Lie algebras of various Lie groups!
   4.5. Calculating 1 parameter subgroups via the exponential homomorphism!
   4.6. Show any discrete normal subgroup of a connected Lie group is central!
   4.7. Show that the universal cover of topological group is a topological group (you can assume it exists)!
5. I am not promising that I will restrict myself to this material, but if you feel confident about the above then you should have no problems.

References