INTRODUCTION TO ∞-CATEGORIES
EXERCISE SHEET 3

(1) Prove the following equalities:
   (i) $\Delta^n * \partial\Delta^0 = \partial_{n+1}\Delta^{n+1}$ - as subobjects of Δ^{n+1}.
   (ii) $\Delta^m * \partial_i\Delta^n = \partial_{m+i+1}\Delta^{m+n+1}$ - as subobjects of Δ^{m+n+1}.
   (iii) $\Delta^0 * \partial\Delta^n = \Lambda_0^{n+1}$ - as subobjects of Δ^{n+1}.

(2) Carefully verify that there is a natural isomorphism
$$(X^o * Y^o) \cong (Y * X)^o$$
for all simplicial sets X and Y.

(3) Use duality to prove that $u \circledast v$ is right anodyne if v is anodyne. (Hint: recall from the lectures that $u \circledast v$ is left anodyne if u is anodyne.)

(4) Give examples of pushout joins of monomorphisms which (a) are not inner anodyne, (b) are left anodyne but not inner anodyne, (c) are anodyne but not left anodyne, and (d) are not anodyne.

(5) Show that if $u : A \to B$ is a monomorphism, then $u * \mathrm{id} : A * \Delta^0 \to B * \Delta^0$ is right anodyne. Conclude that the canonical inclusion $\Delta^0 \hookrightarrow B * \Delta^0$ is right anodyne for any simplicial set B. Is it also inner anodyne?

(6) Show that the nerve functor preserves the slice operation, i.e., given a functor $f : C \to D$, there is a natural isomorphism $N(D/f) \cong N(D)/N(f)$. (D/f is the category of cones with base f).