BLOCK COURSE ON SPECTRAL SEQUENCES

JUSTIN NOEL

1. Schedule

Talk schedule:

(1) Thursday October 6th
  (a) 10:00-10:45 Oliver Trinkaus (topic required!)
  (b) 10:50-11:35 Maximilian Blml (topic required)
  (c) 12:25-13:10 Oliver Rmpelein (topic?)
  (d) 13:15-14:00 Benedikt Preis (Tor groups)
(2) Friday October 7th
  (a) 10:00-10:45 Gesina Schwalbe (Yoneda ext and extensions)
  (b) 10:50-11:35 Julian Seipel (Massey products)
  (c) 12:25-13:10 Jonathan Glckle (Čech cohomology)
  (d) 13:15-14:00 Johannes Loher (Grothendieck spectral sequence).

Here is a list of meeting times (in M206):

(1) Monday, September 26th
  (a) 9:30-10:30 Oliver Trinkaus
  (b) 10:30-11:30 Jonathan Glckle
  (c) 12:30-13:30 Julian Seipel
  (d) 13:30-14:30 Maximilian Blml
(2) Tuesday, September 27th
  (a) 9:30-10:30 Oliver Rmpelein
  (b) 10:30-11:30 Benedikt Preis
  (c) 12:30-13:30 Gesina Schwalbe
  (d) 13:30-14:30 Johannes Loher

2. Talk topics

Unless otherwise stated, you will generally be expected to supply proofs for claims made during your talk. If a text makes a claim without proof, then you need to supply your own proof. Be especially aware of the phrases: “It is easy to see”, “clearly”, and “obviously”. Organize your talk into definitions and stated theorems, propositions, and lemmas. For the proofs and examples make sure that *every* claim is justified.

If the proof for a claim seems to take much longer than you have time for then bring this to the attention of the instructor *before* your talk is expected to be ready.

(1) Projective resolutions (Bachelor's level).
  (a) [Wei94, §2.2] (Skip 2.2.8).
  (b) Do exercises as time permits (include in write-up).
(2) Injective resolutions (Bachelor's level).
  (a) [Wei94, §2.3] (Recall statement of Zorn's lemma before proceeding to Baer's Criterion 2.3.1).
  (b) Do everything through Exercise 2.3.4.

   (c) Lemma 2.3.4 is crucial and needs to be proven. This will be combined with results from [Wei94, §2.3] to prove the results following Lemma 2.3.4.
(3) First calculations in Tor-groups (Bachelor's level).
   (a) [Wei94, §3.1] including all exercises.
   (b) Make sure to give the definition of the Tor-groups first (and why they are well-defined).
(4) First calculations in Ext-groups (Bachelor's level).
   (a) [Wei94, §3.3] including all exercises.
   (b) Make sure to give the definition of the Ext-groups first (and why they are well-defined).
(5) (Gesina Schwalbe) Yoneda Ext and extensions.
   (a) The goal is to complete [Eis95, Exer. A3.26-A.3.27].
   (b) [Eis95, §A.3.11] [Wei94, §3.4].
(6) (Benedikt Preis) Calculating Tor groups over a local ring.
   (a) Prove [Eis95, Cor. A.3.23] via [Eis95, Exer. A3.42-A3.43].
(7) (Bachelor) Computations with the Serre spectral sequence: $H_*(\Omega S^n; k), H^*(\Omega S^n; k)$ including multiplicative structure (at least for cohomology).
   (a) This talk is primarily an exercise building on [McC85, Ex. 1.H, pp. 156-159]
   (b) In terms of increasing difficulty one should try this with $k \in \{\mathbb{Q}, \mathbb{F}_2, \mathbb{F}_p \text{ p odd}, \mathbb{Z}\}$. Present the most general calculation achieved.
   (c) If possible identify the Hopf algebra structure and duality.
(8) Computations with the Serre spectral sequence: $H^*(SU(n))$, $H^*(U(n))$, $H^*(Sp(n))$.
   (a) Work through [McC85, Ex. 5.4].
   (b) Explain all claims in detail.
(9) Serre spectral sequence for wreath products.
   (a) [AM04, pp. 116-119].
(10) Computations with the Lyndon-Hochschild-Serre spectral sequence: $H^*(BD_8)$.
   (a) [AM04, §IV.2].
(11) (Johannes Loher) The Grothendieck composite functor spectral sequence.
   (a) Formulate and prove the existence of this spectral sequence via the spectral sequence associated to a double complex [Wei94, §5.8].
   (b) Fill out [Wei94, Applications 5.8.5].
(12) (Jonathan Glöckle) Comparison between Čech and de Rham cohomology (Bachelor's level).
   (a) The goal of this talk is to introduce the spectral sequences associated to a double complex and deduce [BT95, Ex. 14.16] as a corollary. Bott and Tu spend much more time talking about this argument then you will have for your talk. You may find the treatment in [McC85, §2.4] a little bit more straightforward.
   (b) Because Čech cohomology and de Rham cohomology are not part of this course, you will have to give a *minimal* introduction to these concepts. Simple definitions and some unproven properties should suffice. You should practice to keep the time for this segment short.
   (c) As time permits you should show how the same type of argument applies to show that one can calculate $\mathrm{Tor}_R(A, B)$ by taking a projective resolution of either A or B [McC85, Prop. 2.17].
(13) (Julian Seipel) Massey products.
   (a) [McC85, §8.2, p. 302-307].
   (b) There are two choices here:

(i) (Bachelor's level) The algebraic approach: Define (higher order) Massey products and connect them to spectral sequences by proving the 'staircase lemma' and [McC85, Thm. 8.31]. Be careful about the indeterminacy of these operations!

(ii) (Master's level)The geometric approach: Define the Massey triple product carefully and relate it to the Whitehead product in the universal example [McC85, Thm. 8.27]. Recall the relationship between linking numbers and cup products via Alexander duality. Follow [Mas69, §4]=[Mas98, §4] for the application to Borromean rings.
Basics about Whitehead products can be found in [Whi78], while Alexander duality can be found in many standard algebraic topology texts such as [Spa81].

References

[AM04] Alejandro Adem and R. James Milgram. *Cohomology of finite groups*, volume 309 of *Grundlehren der Mathematischen Wissenschaften [Fundamental Principles of Mathematical Sciences]*. Springer-Verlag, Berlin, second edition, 2004.

[BT95] Raoul Bott and Loring W. Tu. *Differential Forms in Algebraic Topology*. Number 82 in Graduate Texts in Mathematics. Springer-Verlag, 1 June 1995.

[Eis95] David Eisenbud. *Commutative Algebra with a View Toward Algebraic Geometry*. Springer-Verlag, 1995.

[Mas69] W. S. Massey. Higher order linking numbers. In *Conf. on Algebraic Topology (Univ. of Illinois at Chicago Circle, Chicago, Ill., 1968)*, pages 174–205. Univ. of Illinois at Chicago Circle, Chicago, Ill., 1969.

[Mas98] W. S. Massey. Higher order linking numbers. *J. Knot Theory Ramifications*, 7(3):393–414, 1998.

[McC85] John McCleary. *User's guide to spectral sequences*, volume 12 of *Mathematics Lecture Series*. Publish or Perish Inc., Wilmington, DE, 1985.

[Spa81] Edwin H. Spanier. *Algebraic topology*. Springer-Verlag, New York, 1981. Corrected reprint.

[Wei94] Charles A. Weibel. *An Introduction to Homological Algebra*. Cambridge University Pres, 1994.

[Whi78] George W. Whitehead. *Elements of homotopy theory*, volume 61 of *Graduate Texts in Mathematics*. Springer-Verlag, New York-Berlin, 1978.